

# The Cap Set Problem and the Polynomial Method

Naz Kazar

October 20, 2026

## Abstract

The cap set problem asks for the maximum size of a subset  $A \subseteq \mathbb{F}_3^n$  containing no nontrivial three-term arithmetic progression. In this talk, we will discuss the polynomial method approach developed by Croot, Lev, and Pach and adapted by Ellenberg and Gijswijt [1], which yields an exponential upper bound for the size of such sets. The argument is based on low-degree polynomials over finite fields, their evaluation on  $\mathbb{F}_3^n$ , and a rank estimate for a matrix arising from these evaluations. This leads to the bound  $|A| \leq 3m_{2n/3}$ , where  $m_d$  denotes the number of reduced monomials in  $n$  variables of total degree at most  $d$ , and consequently to the estimate  $|A| = o(2.756^n)$ . The aim of the talk is to illustrate how polynomial techniques and linear algebra can be used to obtain strong bounds in additive combinatorics.

## References

- [1] Jordan S. Ellenberg and Dion Gijswijt, *On large subsets of  $\mathbb{F}_q^n$  with no three-term arithmetic progression*, Annals of Mathematics (2) 185 (2017), no. 1, 339–343. doi:10.4007/annals.2017.185.1.8.